Hydrodynamic Transport in clean Fermi and Non-Fermi liquids

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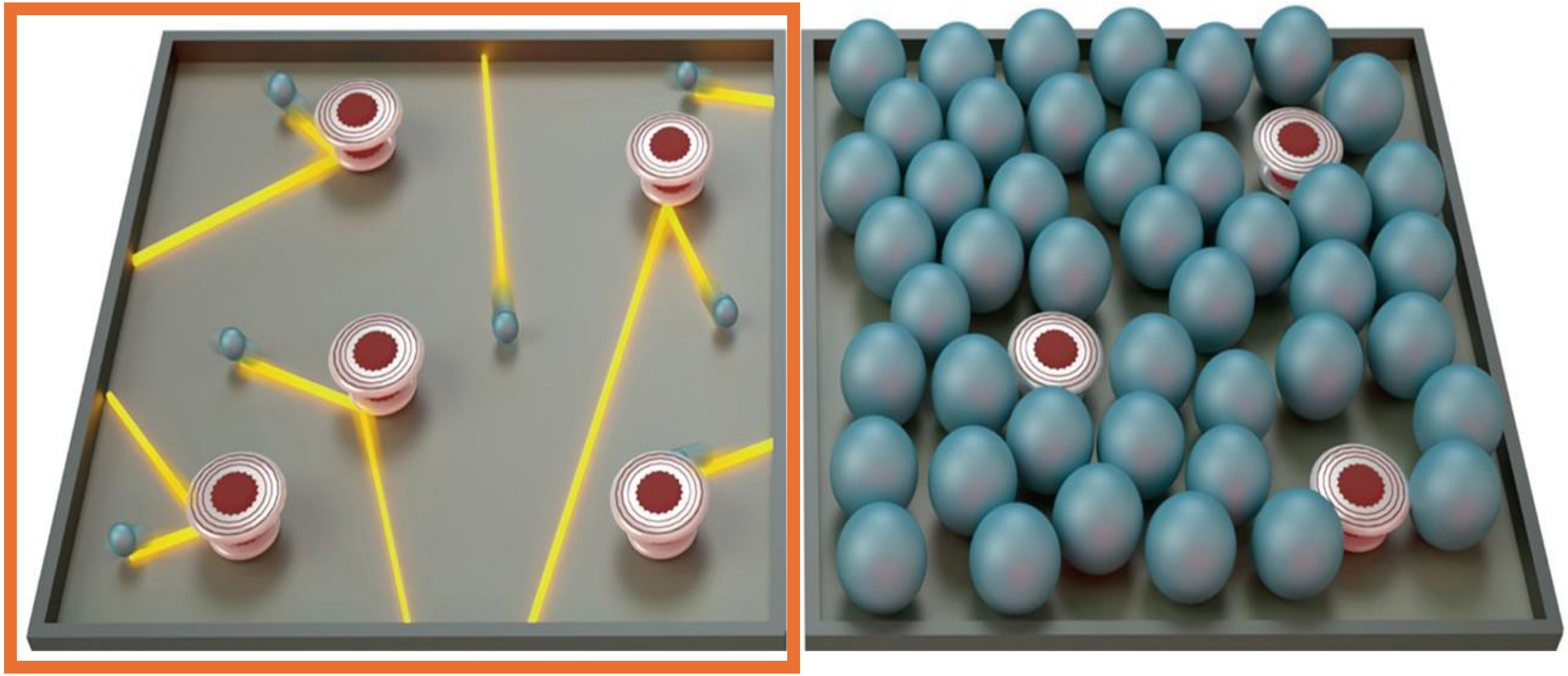
Transport in conventional metal

- DC transport in conventional metal is characterized by the Drude formula

$$\sigma_{xx} = \frac{ne^2}{m}\tau$$

- τ: the timescale for electrons to lose momentum to disorder, lattice or phonons etc.
- In a Fermi liquid dominated by electron-electron Umklapp scattering, $\tau^{-1} \propto A + BT^2$

Conventional transport (free particle)



Disorder dominated: $\gamma_{disorder} \gg \gamma_{ee}$

Jan Zaanen, Science 2016

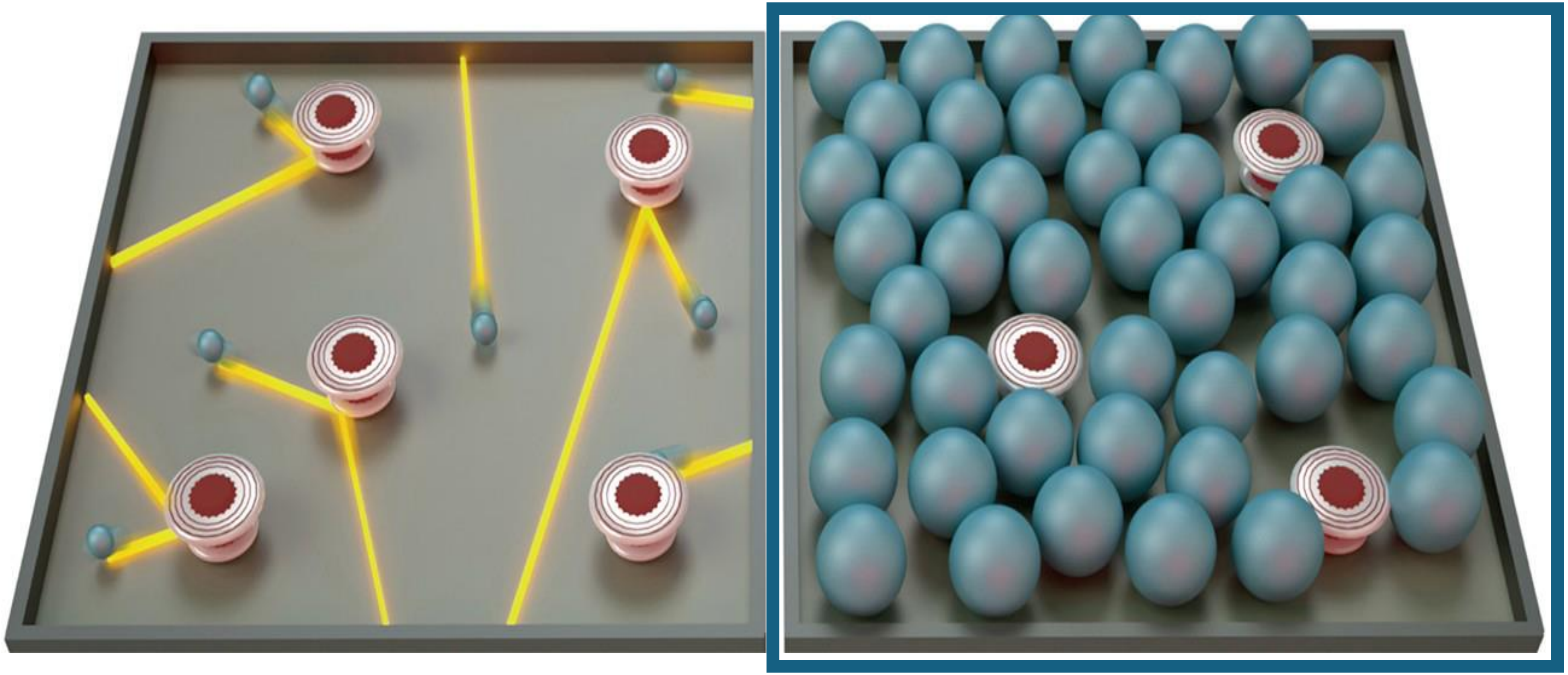
Transport in ultra clean metal

- However, in an ultra clean metal, the total momentum of the electrons is almost conserved (assuming low-temperature and small FS)

- This leads to infinite DC conductivity in the Drude formula $\sigma(\omega) \sim \sigma_0 \delta(\omega)$

- How do we probe a clean metal through transport?

Unconventional transport



Interaction dominated: $\gamma_{disorder} \ll \gamma_{ee}$

Jan Zaanen, Science 2016

Hydrodynamics

- When momentum-conserving electron-electron collisions dominate over other scattering mechanism, hydrodynamics emerge

- The electrons flow is governed by Navier-Stokes equation, just like water

神奈川沖浪裏　葛飾北斎

Photo at Museum of Fine Arts, Boston

Non-Fermi liquid

- In Landau's FL theory, the electron-electron scattering rate ($\gamma_{ee} \sim T^2/E_F$) is still parametrically smaller than the energy of a quasiparticle T

- However, in the presence of strong interaction (e.g. near a quantum critical point), $\gamma_{ee} \gg T$, and the quasiparticle is destroyed

- If the interaction still conserves momentum hydrodynamics should still emerge? How is this different from FL hydrodynamics?

Less disorder

Stronger
Interactio
n

FL, Drude formula	FL, Hydrodynamics
NFL, Drude formula?	?

Outline

- Fermi liquid Hydrodynamics
  - Superballistic conduction in Graphene
  - Tomographic transport and linear-in-T conductance
- Non-Fermi liquid Hydrodynamics
  - Yukawa-SYK model of Ising-Nematic quantum critical point
  - Boltzmann equation as an 1/N effective field theory
  - Instability of Ising-Nematic QCP NFL

Crush course on Hydrodynamic Transport

What is hydrodynamics?

- Hydrodynamics describe the long wavelength physics based on conservation laws

- They are simple to write: Just continuity equations:

$$\partial_\mu j^\mu = 0 \qquad \text{particle conservation}$$

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{momentum conservation}$$

- Combining with constituent relations (E.g. $\vec{j} = ne\vec{v}$), we can close the equations and solve for the dynamics

Ohmic vs Hydrodynamic transport

- Ohmic transport:
  - Local field-current relation (Ohm's law)

$$\vec{j}(\vec{x}) = \sigma \vec{E}(\vec{x})$$

  - Electron momentum is dissipated at every point in the bulk
  - Stronger scattering => Less conductive

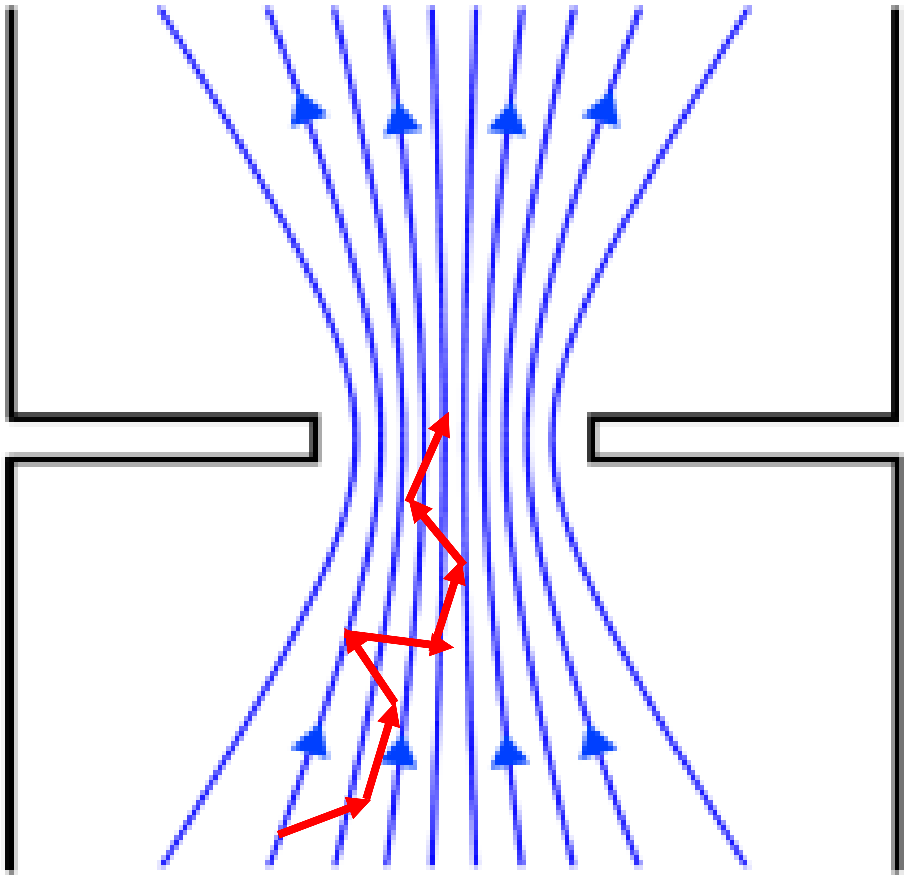
- Hydrodynamic transport
  - Non-local field-current relation (Navier-Stokes Eq)

$$\eta \nabla^2 \vec{j}(\vec{x}) = (ne)^2 \vec{E}(\vec{x})$$

  - Momentum does not dissipate in the bulk. It only happens at the boundary of the system
  - Stronger scattering => More conductive

Interaction as a lubricant

- Bulk scattering does not relax momentum nor the current
- Current is only dissipated at the boundary
- Individual electron is random walking along the stream line
- Stronger scattering => Harder to reach the boundary => Less dissipation



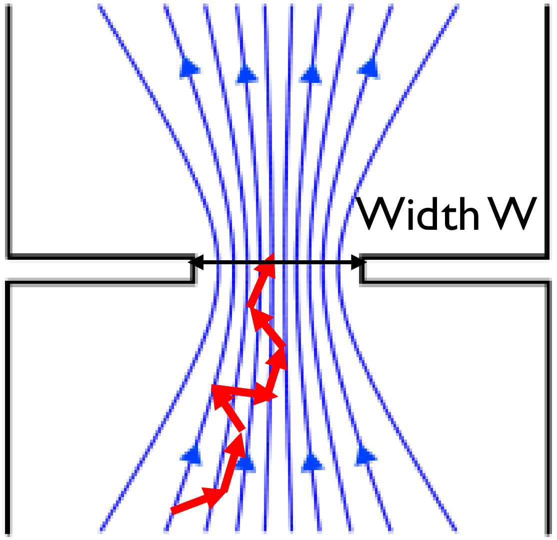
Non-local conductivity and Anomalous Conductance

- The NS equation yields a $\vec{q}$-dependent conductivity:

$$\sigma(\vec{q}) = \frac{n^2 e^2}{\eta |\vec{q}|^2}$$

- Therefore, the conductance G depends on an external length scale W via the relation $G \sim \sigma(1/W)$
  - E.g. in a constriction geometry

$$G_{\text{viscoous}} = \frac{\pi n^2 e^2 W^2}{32\eta}$$



Super-Ballistic conduction!

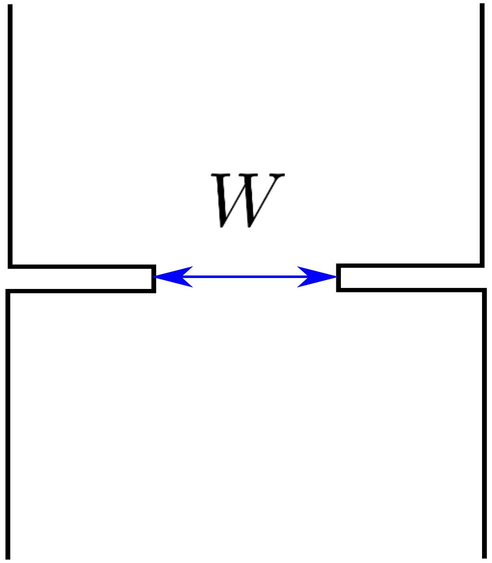
- An interacting metal can be more conductive than a Fermi gas!
- Free particle:
  - Landauer (1957) and Sharvin (1965): Count how many standing waves are supported in the constriction

$$G_{\text{ballistic}} = \frac{2e^2}{h} \times \frac{2W}{\lambda_F}$$

- Viscous fluid:

$$G_{\text{viscoous}} = \frac{\pi n^2 e^2 W^2}{32\eta}$$

  - Can be much larger when W is large



PNAS PNAS

Higher-than-ballistic conduction of viscous electron flows

Haoyu Guo[a], Ekin Ilseven[a], Gregory Falkovich[b,c], and Leonid S. Levitov[a,1]

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LETTERS
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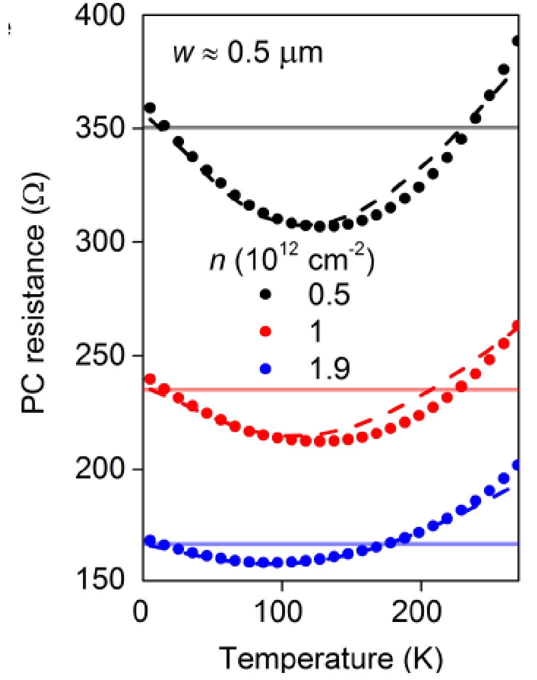
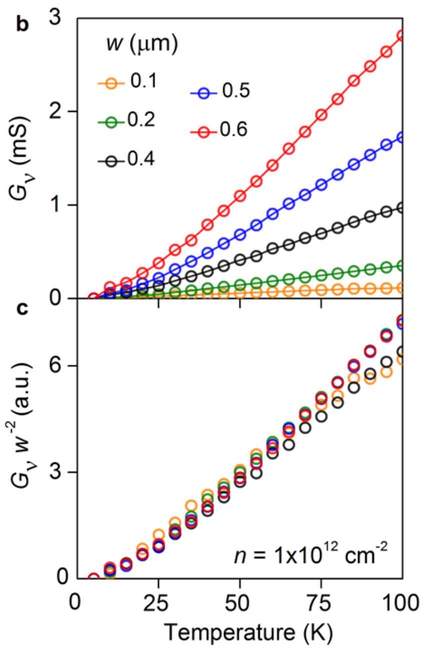
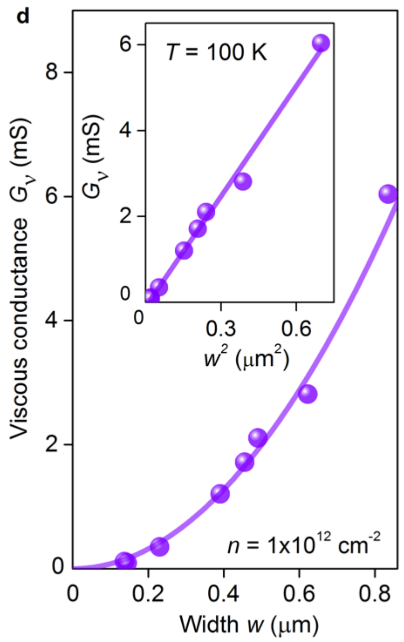
nature physics

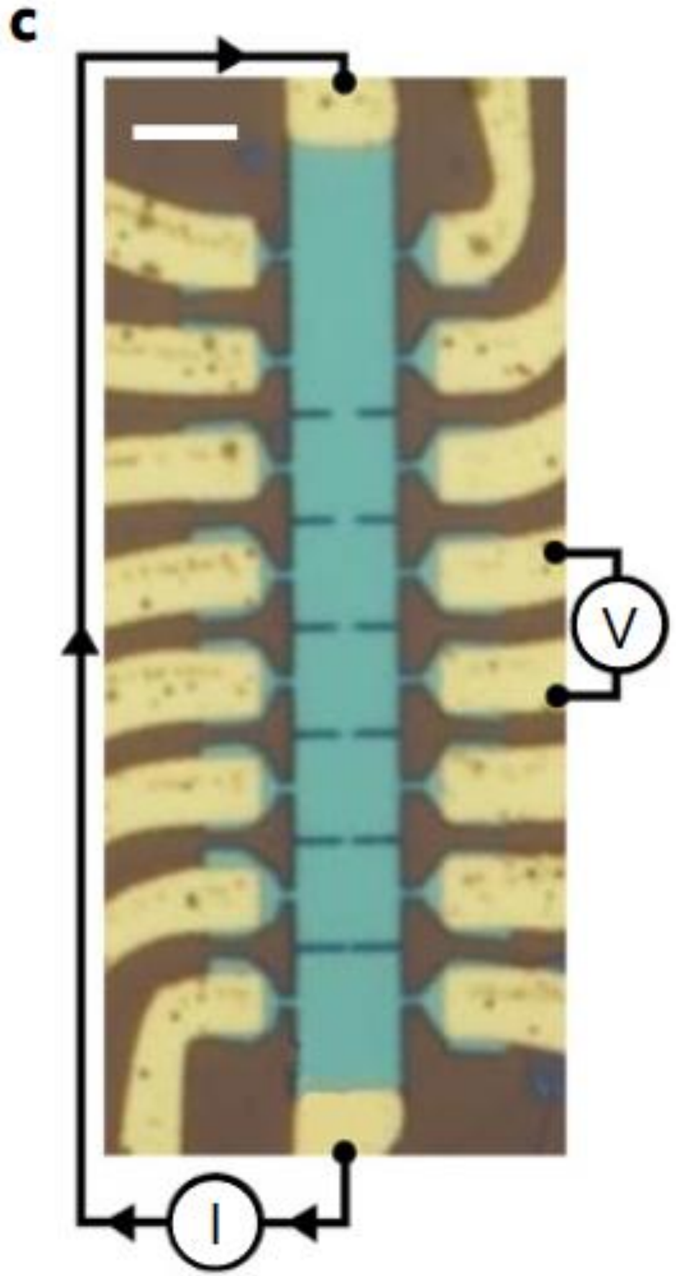
Superballistic flow of viscous electron fluid through graphene constrictions

R. Krishna Kumar[1,2,3], D. A. Bandurin[1,2], F. M. D. Pellegrino[4], Y. Cao[2], A. Principi[5], H. Guo[6], G. H. Auton[2], M. Ben Shalom[1,2], L. A. Ponomarenko[3], G. Falkovich[7,8], K. Watanabe[9], T. Taniguchi[9], I. V. Grigorieva[1], L. S. Levitov[6], M. Polini[1,10] and A. K. Geim[1,2]*



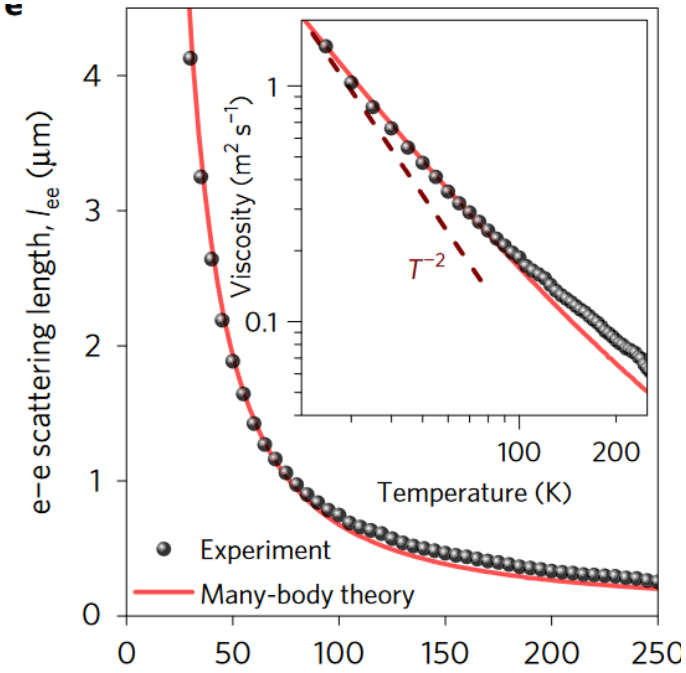






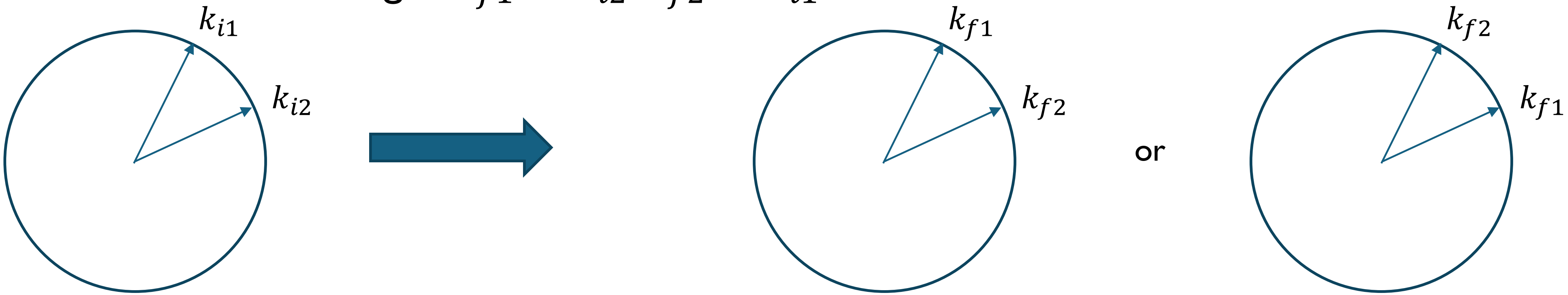
What should the mean-free path be?

- The el-el mean free path can be extracted from $l_{ee} = 4\eta/(n m_e v_F)$
- From FL theory, we would expect $l_{ee} \propto 1/T^2$
- But this does not quite fit the experiment
- What is missing?



Fermiology comes into play

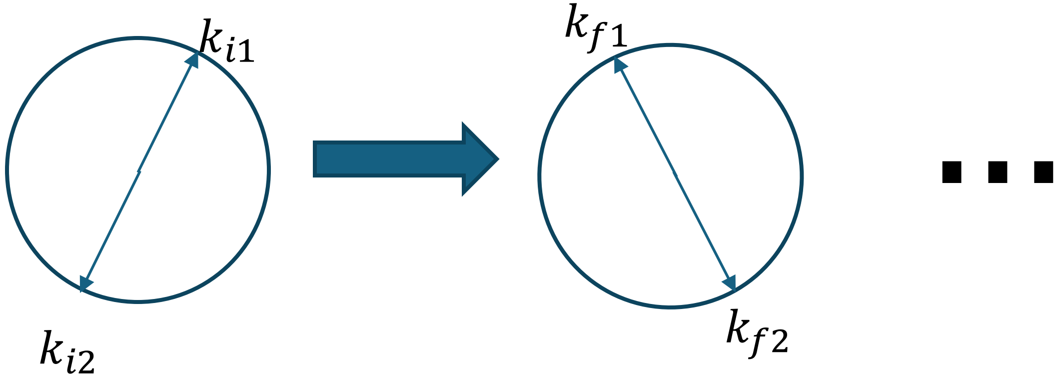
- Fermion scattering is kinematically constrained for 2D convex FS beyond what FL theory requires
- Generic scattering ($k_{i1} + k_{i2} \neq 0$) does not cause dissipation:
  - Only two scattering configurations:
  - Forward scattering: $k_{f1} = k_{i1}, k_{f2} = k_{i2}$
  - Particle exchange: $k_{f1} = k_{i2}, k_{f2} = k_{i1}$



Head-on Scattering $k_{i1} + k_{i2} = 0$

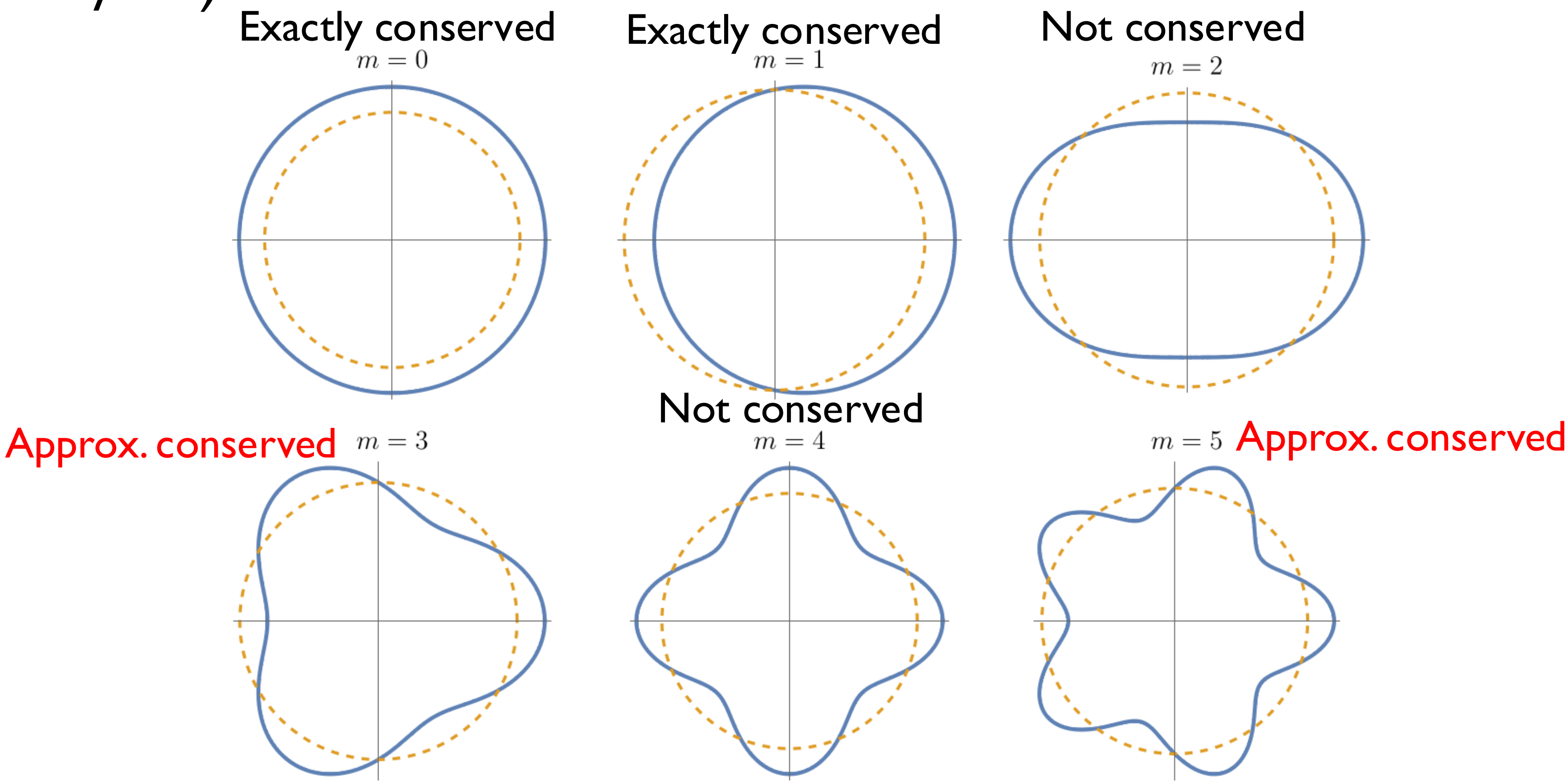
- Any head-on initial pair can scatter to any head-on pair
- However, head-on configurations have even parity, and they can't relax odd parity deformations
- Similar reasonings apply to any convex, inversion-symmetric FS

P. J. Ledwith, HG and L. Levitov, Annals of Physics 411, 167913 (2019)
D. L. Maslov, V. I. Yudson, and A. V. Chubukov, Phys. Rev. Lett. 106, 106403 (2011)
H.K. Pal, V. I. Yudson, D. L. Maslov, Lithuanian Journal of Physics (2012)



Long-lived odd-parity deformation

- Odd parity deformations of the FS are long lived (compared to quasiparticle lifetime $1/T^2$)



Boltzmann description

- These approximately conserved modes can be described by a linearized Boltzmann equation

$$f = f(t, \vec{x}, \theta_k) = \sum_m f_m(t, \vec{x}) e^{im\theta_k}$$

$$\partial_t f + \vec{v}_k \nabla f = -I[f] \qquad \text{Collision operator} \quad I[f_m] = -\lambda_m f_m$$

$$\lambda_0 = \lambda_1 = 0 \quad \text{particle and momentum conservation}$$

$$\lambda_2, \lambda_4, \cdots \sim T^2/E_F \qquad \text{FL theory}$$

$$\lambda_3, \lambda_5, \cdots \ll T^2/E_F \quad \text{approximate conserved modes}$$

Calculating the odd-m rates

- Apply Fermi's Golden rule to the collision term

$$I[f(\boldsymbol{p}_i)] = \int \frac{d^2p_j d^2p_{i'} d^2p_{j'}}{(2\pi)^6}(W_{i'j'\to ij} - W_{ij\to i'j'})$$

$$W_{ij\to i'j'} = \frac{2\pi}{\hbar}|V|^2 f_i f_j (1-f_{i'})(1-f_{j'})\,\delta\left(\sum_\alpha{}' \varepsilon_\alpha\right)(2\pi)^2\delta^{(2)}\left(\sum_\alpha{}' \boldsymbol{p}_\alpha\right)$$

- The decay rates are eigenvalues of linearized $I[\cdot]$

$$\lambda_m^{\mathrm{odd}} \sim \frac{m^4T^4}{E_F^3}\ln m$$

P. J. Ledwith, HG and L. Levitov, Annals of Physics 411, 167913 (2019)

Conductivity

- Solving the Boltzmann Eq., we obtain a scale-dependent viscosity

$$\sigma(k) = \frac{n^2 e^2}{k^2 \eta(k)} \qquad \eta(k) = \frac{n m_e v_F^2}{4\Gamma_2(k)}$$

- The effective scattering rate is a continuous fraction:

$$\Gamma_2(k) = \lambda_2 + \frac{k^2 v_F^2/4}{\lambda_3 + \frac{k^2 v_F^2/4}{\lambda_4 + \ldots}}$$

P. Ledwith, HG, A. Shytov, Leonid Levitov, PRL 2019
Serhii Kryhin, Qiantan Hong and Leonid Levitov, arXiv:2310.08556

Two regimes of hydrodynamics

$$\Gamma_2(k) = \lambda_2 + \frac{k^2 v_F^2/4}{\lambda_3 + \frac{k^2 v_F^2/4}{\lambda_4 + \ldots}}$$

- Conventional Hydro ($k \to 0$)
  - $\Gamma_2 = \lambda_2 \propto T^2$, should agree with FL
- Tomographic transport ($k > l_{tomo}^{-1}$)
  - We need to perform a non-perturbative summation of Γ_2
  - $\Gamma_2(k) \propto k^{1/3} T$
  - Experimental implications:
    - Happens in a small device (shorter than l_{tomo})
    - Modifying $G \propto W^2$ to $G \propto W^{\frac{5}{3}}$ (Hard to test)
    - $G \propto T$, linear-in-temperature conductance in a FL!

P. Ledwith, HG, A. Shytov, Leonid Levitov, PRL 2019
Serhii Kryhin, Qiantan Hong and Leonid Levitov, arXiv:2310.08556

arXiv:2407.05026

Quantitative measurement of viscosity in two-dimensional electron fluids

Yihang Zeng[1,5], Haoyu Guo[2,4], Olivia M. Ghosh[1], Kenji Watanabe[3],
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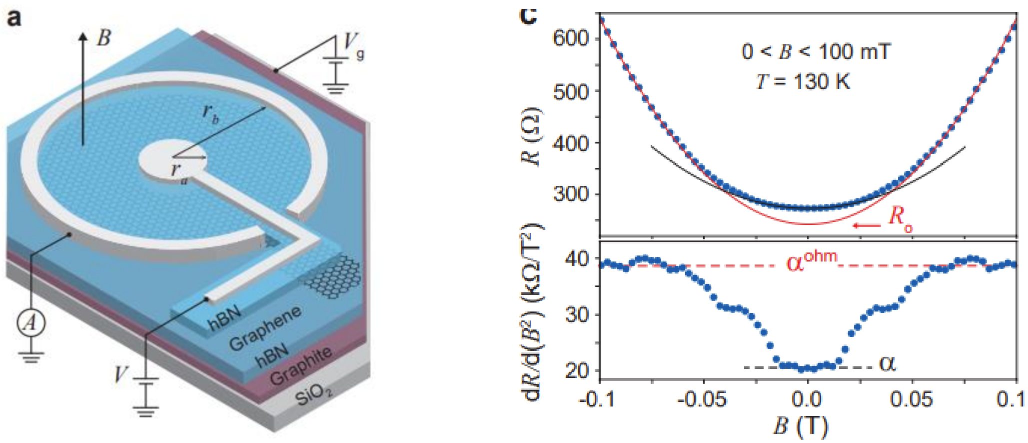
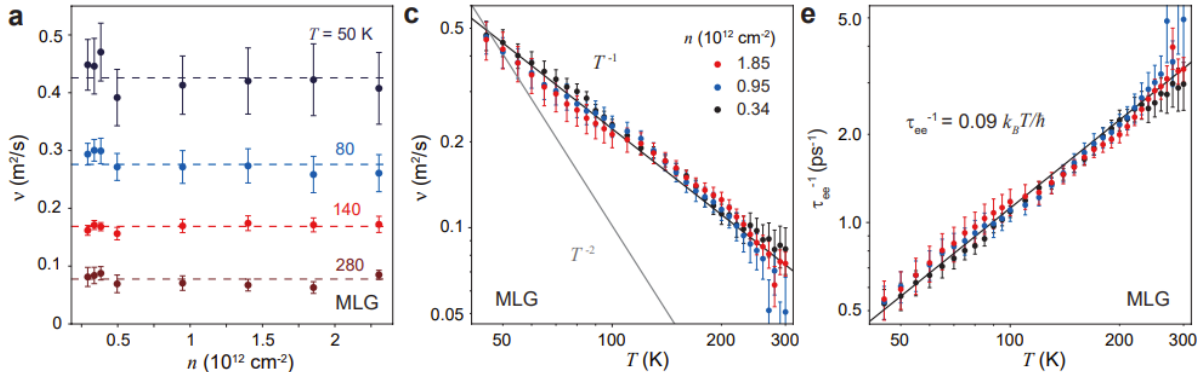
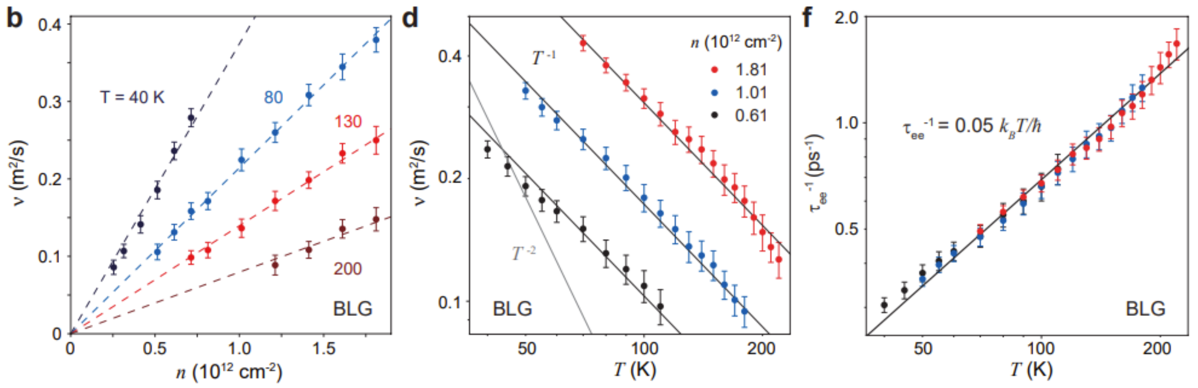
[3] National Institute for Materials Science, Tsukuba, Japan
[4] Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA and
[5] Present address: Department of Physics, Purdue University, West Lafayette, Indiana, USA
(Dated: July 9, 2024)

- Measure viscosity through magneto transport
- Allows subtraction of phonon background => Access larger temperature range
- Find linear-in-T scattering rate





Monolayer



Bilayer

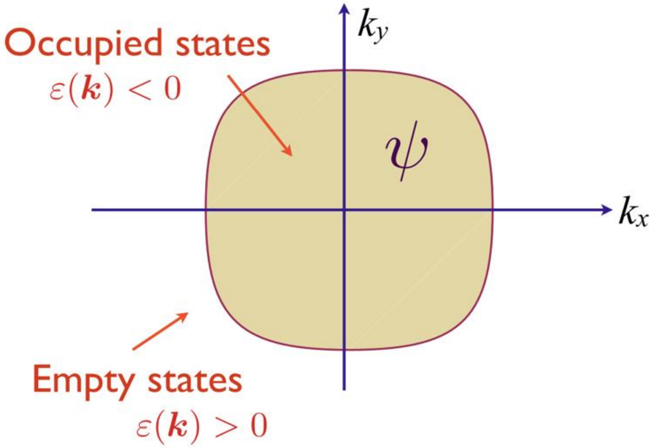
Fluctuation Spectrum of Non Fermi-Liquids

HG, arXiv:2406.12967

Critical FS

- We study the toy model of Non-Fermi liquid: A Fermi surface coupled to a gapless dynamical boson (e.g. at a quantum critical point) at Q=0
- The model can be schematically written as

$$\mathcal{L} = \psi^\dagger(\partial_\tau + \hat{\varepsilon}_k)\psi + \frac{1}{2}\phi(\partial_\tau^2 + q^{z_b - 1} + r)\phi + g\phi\psi^\dagger\psi$$



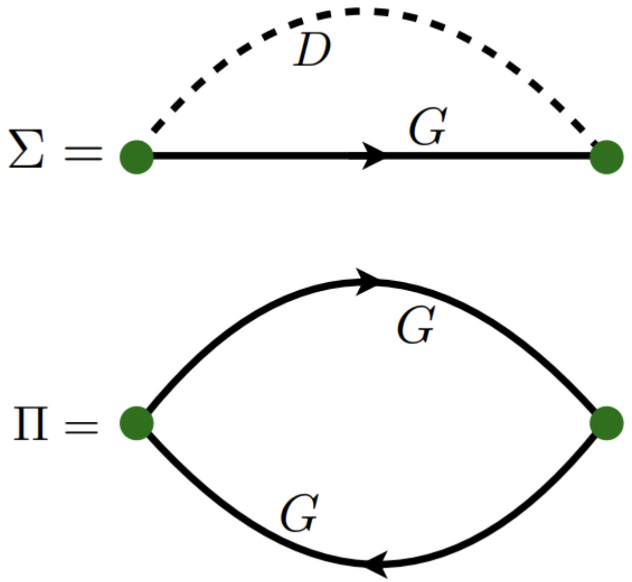
➕

A critical boson ϕ

- Ising-Nematic order
- Ferrormagnetic order
- Transverse component of abelian or non-abelian gauge field

The Migdal-Eliashberg Theory

- In many literature, the leading order physics of the critical FS is described by the Migdal-Eliashberg Theory: Vertex-corrections are ignored in the computation self-energies



$$\text{Exactly at the QCP}$$

$$\Sigma(i\omega, k) = -i\mathrm{sgn}(\omega)|\omega|^{2/z_b}\omega_P^{1-2/z_b}$$

$$\Pi(i\Omega, q) \propto -\frac{|\Omega|}{q}$$

$$\text{NFL below scale } \omega_P\text{: } \Sigma(i\omega) \propto |\omega|^{2/z_b} \gg |\omega|$$

$$\text{Local bosonic Hamiltonian} \to z_b = 3$$

Can the FL story generalize?

- The FL story depends on the Boltzmann equation, which is conventionally justified with the quasiparticle concept
- In a NFL without quasiparticle, there can be alternative Boltzmann formulation due to Prange and Kadanoff (1964), by projecting distribution function onto the FS
- Recently, there has been interest in utilizing the Boltzmann equation as a starting point for bosonization of FL and NFL

Bosonization of Non-Fermi Liquids

SangEun Han, Félix Desrochers, and Yong Baek Kim

Nonlinear bosonization of Fermi surfaces: The method of coadjoint orbits

Luca V. Delacrétaz,[1,2] Yi-Hsien Du,[1] Umang Mehta,[1,3] and Dam Thanh Son[1,2,4]

Effective field theory of Berry Fermi liquid from the coadjoint orbit method

Xiaoyang Huang[*]

Coadjoint-orbit effective field theory of a Fermi surface in a weak magnetic field

Mengxing Ye[1,*] and Yuxuan Wang[2,†]

Yukawa-SYK model

- The Eliashberg theory can be made into a systematic large-N expansion using Yukawa-SYK model

$$\mathcal{L} = \sum_i^N \psi_i^\dagger(\partial_\tau + \hat{\varepsilon}_k)\psi_i + \sum_l^N \frac{1}{2}\phi_l(\partial_\tau^2 + q^{z_b - 1} + m_b^2)\phi_l + \sum_{ijl}^N \frac{g_{ijl}}{N}\phi_l \psi_i^\dagger \psi_j$$

- After averaging, the theory can be rewritten in terms bi-local variables

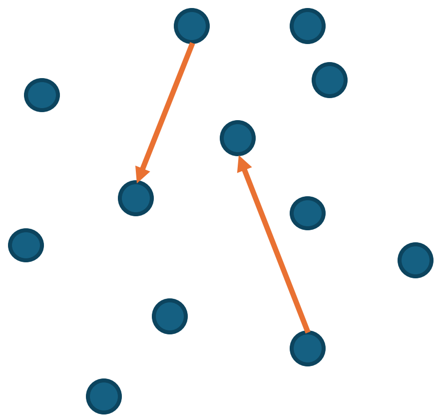
$$\frac{1}{N}S[G,\Sigma,D,\Pi] = -\ln\det\left((\partial_\tau + \varepsilon_k - \mu)\,\delta(x-x') + \Sigma\right) + \frac{1}{2}\ln\det\left(\left(-\partial_\tau^2 + \omega_q^2 + m_b^2\right)\delta(x-x') - \Pi\right)$$
$$-\ \mathrm{Tr}\ (\Sigma \cdot G) + \frac{1}{2}\mathrm{Tr}\ (\Pi \cdot D) + \frac{g^2}{2}\mathrm{Tr}\ ((GD)\cdot G)\ .$$

- The large-N saddle point is the Eliashberg equation

Ilya Esterlis, J. Schmalian, PRB 100, 115132 (2019)
Yuxuan Wang and A.V. Chubukov, PRR 2, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, arXiv:2012.00763
Ilya Esterlis, Haoyu Guo, Aavishkar Patel, Subir Sachdev PRB 103, 235129 (2021)
Zhengyan Darius Shi, Dominic V. Else, Hart Goldman, T. Senthil, Scipost 14,113(2023)

1/N Fluctuation = low-energy theory

- E.g. in the 0+1D SYK model, the 1/N effective theory turns out to be a black hole described by Schwarzian action



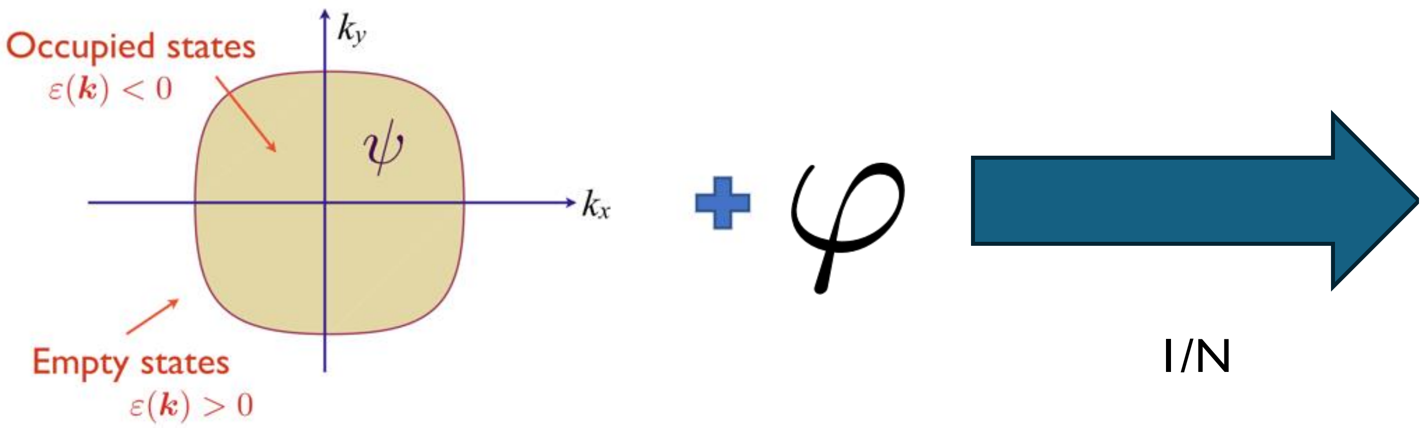
$$H_{\mathrm{SYK}} = \frac{1}{(2N)^{3/2}} \sum_{abcd} U_{abcd} c_a^\dagger c_b^\dagger c_c c_d$$

1/N

$$\frac{S}{N} = -\frac{\alpha_S}{\mathcal{J}} \int d\tau \{\tan \frac{\pi g(\tau)}{\beta}, \tau\} = \frac{\alpha_S}{2\mathcal{J}} \int d\tau \left[\left(\frac{g''}{g'}\right)^2 - \left(\frac{2\pi}{\beta}\right)^2 (g')^2 \right]$$

Emergence of Boltzmann equation

- In the Yukawa-SYK model, we recover the Boltzmann equation as 1/N fluctuation



1/N

$$S_{\text{Eliashberg}} = -\frac{N}{2}\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\int \frac{\mathcal{N}\mathrm{d}\theta_k}{2\pi}\left(\Omega\phi(\theta_k;-p)\right) \times \left(\Omega + i\boldsymbol{v}_k\cdot\boldsymbol{p} + \lambda_{-i\partial_{\theta_k}}\right)\phi(\theta_k;p)\,,$$

HG, arXiv:2406.12967

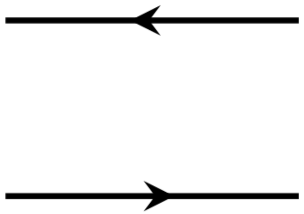
The Gaussian fluctuations

- The leading-order 1/N correction are the Gaussian fluctuations of the bilocal fields

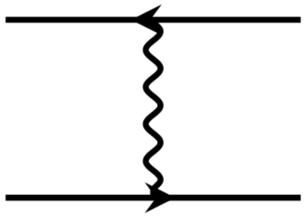
$$\delta^2 S = \frac{N}{2} \int_{x_1, x_2, x_3, x_4} \delta G(x_2, x_1) K_{\mathrm{BS}}(x_1, x_2; x_3, x_4) \delta G(x_3, x_4)$$

- The Bethe-Salpeter kernel can be represented by Feynman diagrams

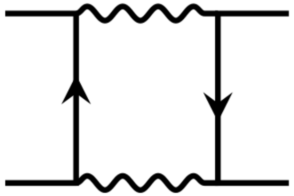
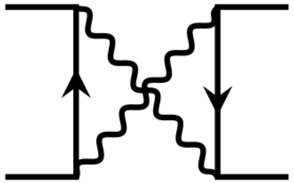
$$K_{\mathrm{BS}} = W_{\Sigma}^{-1} - W_{\mathrm{MT}} - W_{\mathrm{AL}}$$

$W_{\Sigma} =$ 

Density-of-States

$W_{\mathrm{MT}} =$ 

Maki-Thompson

$W_{\mathrm{AL}} =$  $+$ 

Aslamazov-Larkin

Diagonalizing the kernel

- To identify the low-energy fluctuations, we need to diagonalize the kernel K_{BS}
- The first step is to define a good inner product on the space of two-point functions

$$\langle \delta G_1 | \delta G_2 \rangle$$

- This is equivalent to apply a suitable precondtioner M, so that we are diagonalizing $L = K_{BS} \circ M$
- The problem is setup so that L has zero modes associated with conservation laws

$$L[1] = 0 \qquad \text{particle conservation}$$

$$L[\vec{k}] = 0 \qquad \text{momentum conservation}$$

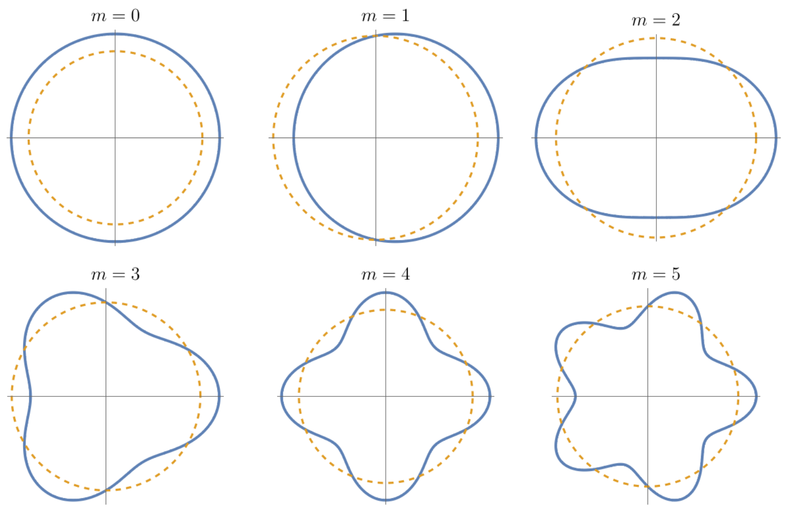
Hierarchy of L

- We assume circular FS and consider $L \to L^{(m)}$ in the m-th angular harmonic channel
- We perform a double expansion of L in
  - Proximity to the FS $\xi/k_F v_F$
  - Small scattering angle $\theta \sim q/k_F$

$$\begin{array}{lccccccc}
 & & \text{On FS} & & O(\frac{\xi}{k_F v_F})\text{ away from FS} & & O((\frac{\xi}{k_F v_F})^2)\text{ away from FS} & \\
 & L_m = & L_m^{(0)} & + & L_m^{(1)} & + & L_m^{(2)} & + \dots \\
 & & \| & & \| & & \| & \\
\text{Forward Scattering:} & & \delta_q^0 L_m^{(0)} & & \delta_q^0 L_m^{(1)} & & \delta_q^0 L_m^{(2)} & \\
 & & + & & + & & + & \\
O(q^2)\text{ small angle scattering:} & & \delta_q^1 L_m^{(0)} & & \delta_q^1 L_m^{(1)} & & \delta_q^1 L_m^{(2)} & \\
 & & + & & + & & + & \\
 & & \vdots & & \vdots & & \vdots &
\end{array}$$

Large number of zero modes at the leading order

- At zero CoM momentum $p = (i\Omega, 0)$, the leading order term $\delta_q^0 L_m^{(0)}$ contains a large number of soft modes
  - $\delta_q^0 L_m^{(0)}[F] = 0, F = \phi(\theta_k)$ is a function of angle on the FS only (indepdent of ξ, ω)
  - F parameterizes the deformation of the FS
- $\delta_q^0 L_m^{(0)}$ describes forward scattering limit, which naturally preserves the FS shape



Effective theory of the soft modes

- Functions of the form $F(i\omega, \xi, \theta) = \phi(\theta)$, are soft modes of the kernel L.
- The soft eigenvalues λ_m can be obtained by performing eigenvalue perturbation theory (details later)
- The low-energy effective theory can be obtained by projecting the action onto the soft mode manifold

$$S_{\mathrm{Eliashberg}} = -\frac{N}{2} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathcal{N} \mathrm{d}\theta_k}{2\pi} \left(\Omega \phi(\theta_k; -p)\right) \times \left(\Omega + i \boldsymbol{v}_k \cdot \boldsymbol{p} + \lambda_{-i\partial_{\theta_k}}\right) \phi(\theta_k; p) \,,$$

- The EoM of this action is exactly Boltzmann equation, where λ plays the role of collision term

The collision rates

- The even-m collision rates are suppressed from the self-energy by the smallness of scattering angle

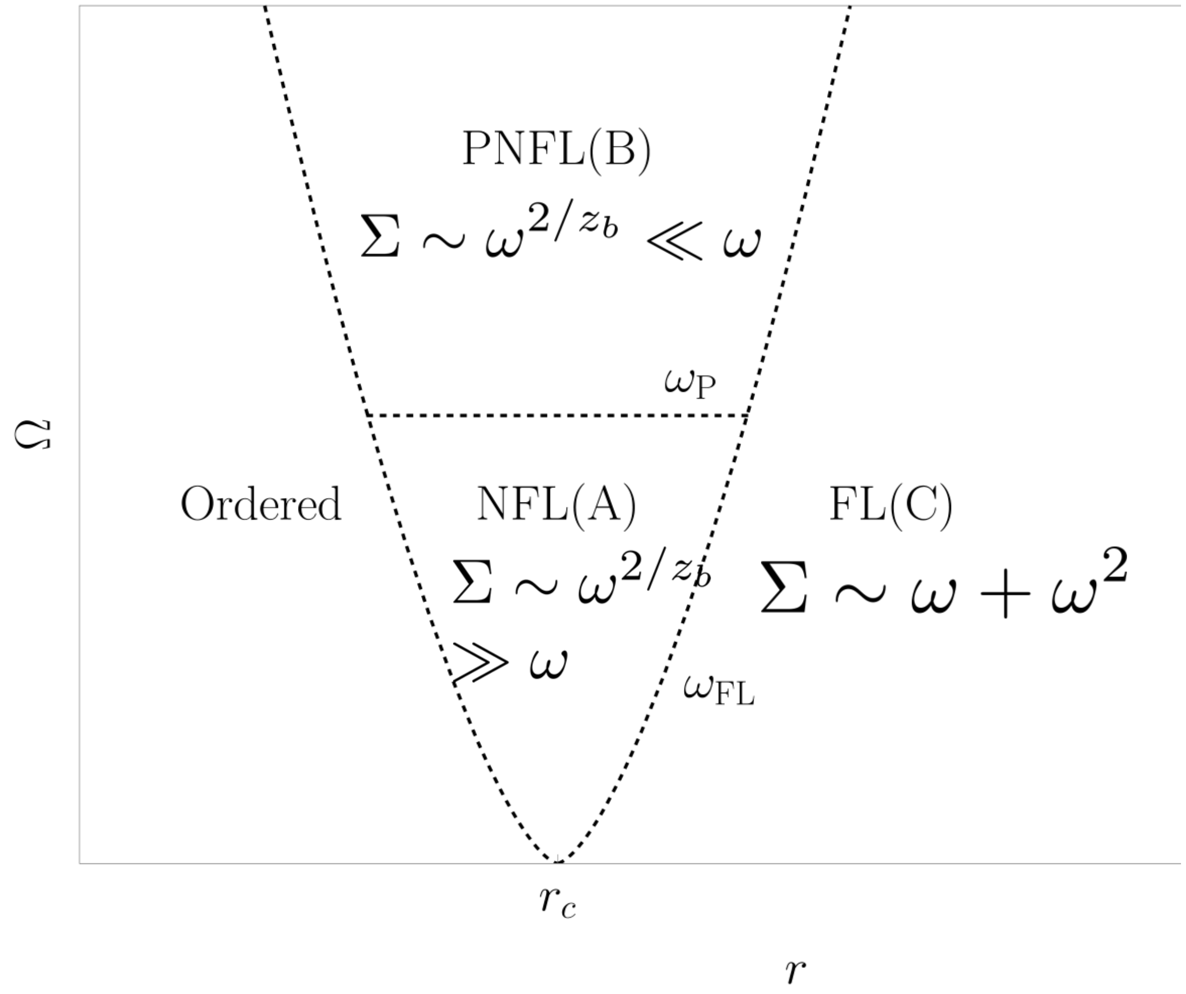
$$\lambda_m^{\mathrm{even}} = \frac{\langle 1|\delta_q^1 L_m^{(0)}|1\rangle}{\langle 1|1\rangle} \propto \underbrace{c_f \Omega^{2/z_b}}_{\text{Self-Energy}} \times \underbrace{\frac{m^2 \langle q^2\rangle}{k_F^2}}_{\text{Small-angle scattering}}$$

- Similar to FL, the odd-m rates are further suppressed due to convexity of the FS

$$\lambda_m^{\mathrm{odd}} \propto \underbrace{\Omega^{2/z_b}}_{\text{Self-Energy}} \times \underbrace{m^2(m^2-1)\frac{\langle \xi^2\rangle}{k_F^2 v_F^2}}_{\text{2nd order perturbation}} \times \underbrace{(m^2-1)\frac{\langle q^2\rangle}{k_F^2}}_{\text{Small-angle scattering}}$$

Different regimes near Ising-Nematic QCP

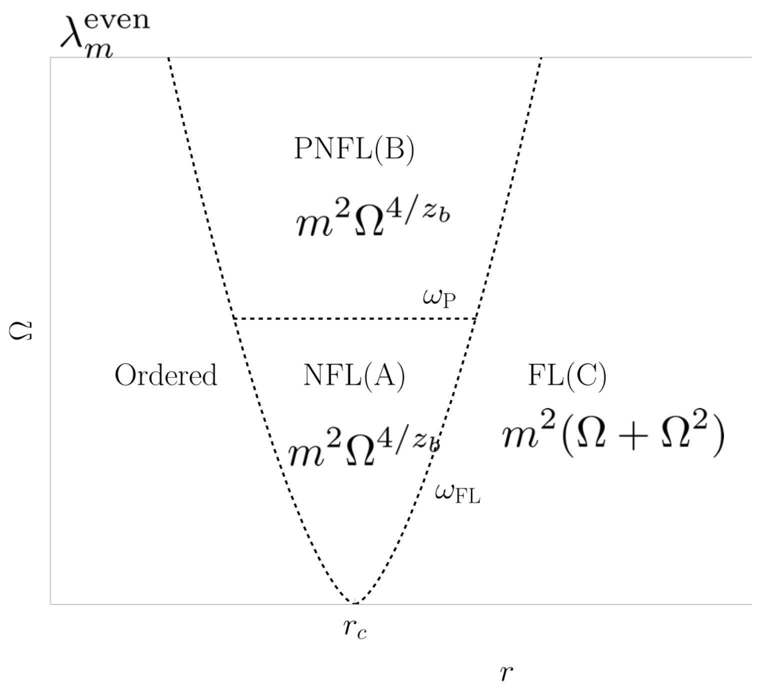
- r is the tuning parameter (boson mass term)
- Region A: quantum critical NFL, $\Sigma \propto \omega^{2/z_b} \gg \omega$
- Region B: perturbative NFL with $\Sigma \propto \omega^{2/z_b} \ll \omega$
- Region C: boson is gapped, but mediates small angle scattering

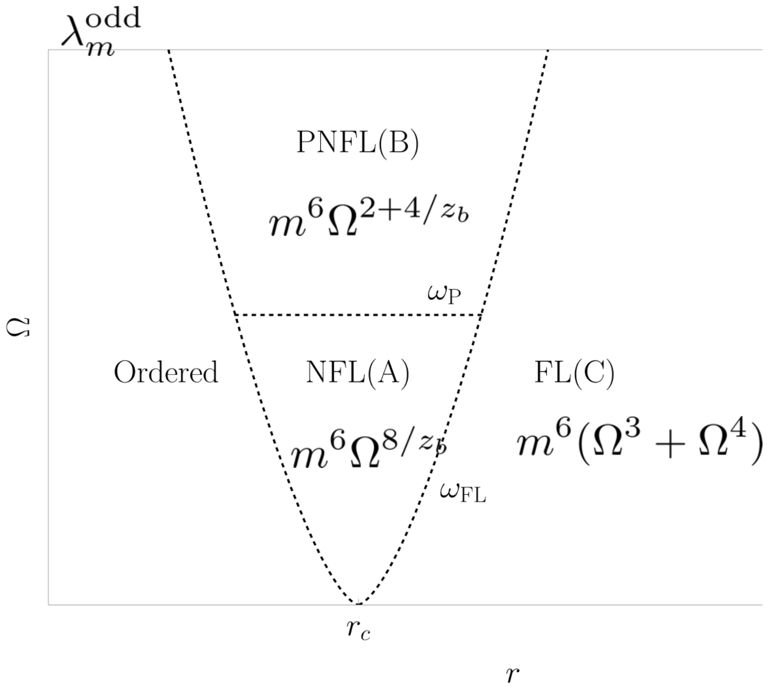


Estimating the collision rates

- The collision rates can be estimated by substituting appropriate scalings for q and ξ at the QCP

$$q \sim \Omega^{1/3} \quad \xi \sim \Omega \text{ when } \Omega \gg \Sigma(i\Omega) \qquad \xi \sim \Omega^{2/3} \text{ when } \Omega \ll \Sigma(i\Omega)$$



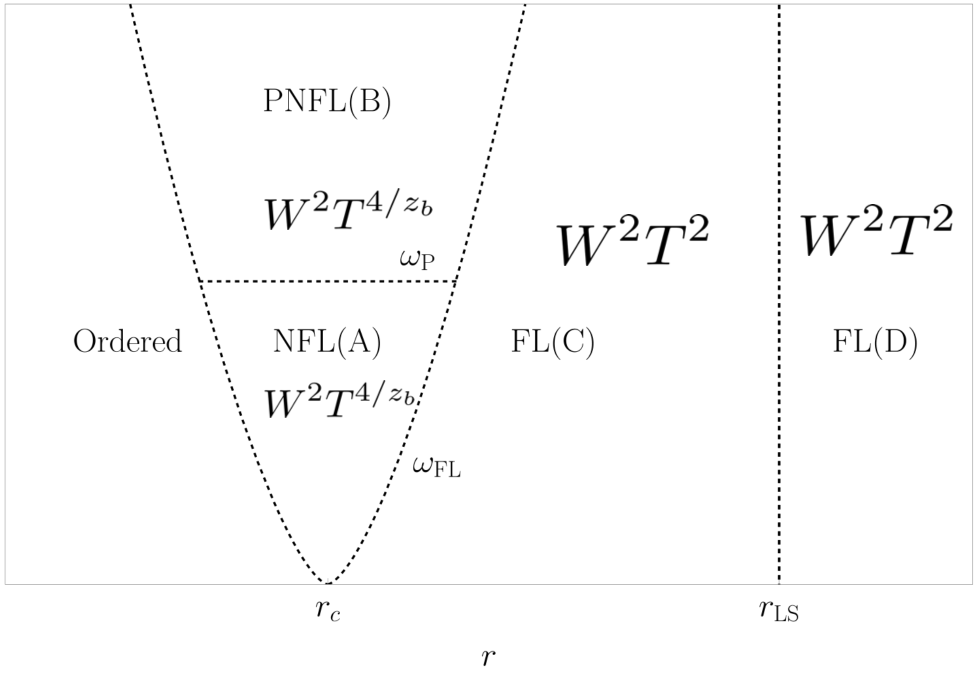


NFL Hydrodynamics

- The hydrodynamics of the NFL can then be obtained from the Boltzmann formalism, with the NFL collision rates

G(T,W) in conventional hydro regime

$l_{ee} < l_{tomo} < W$

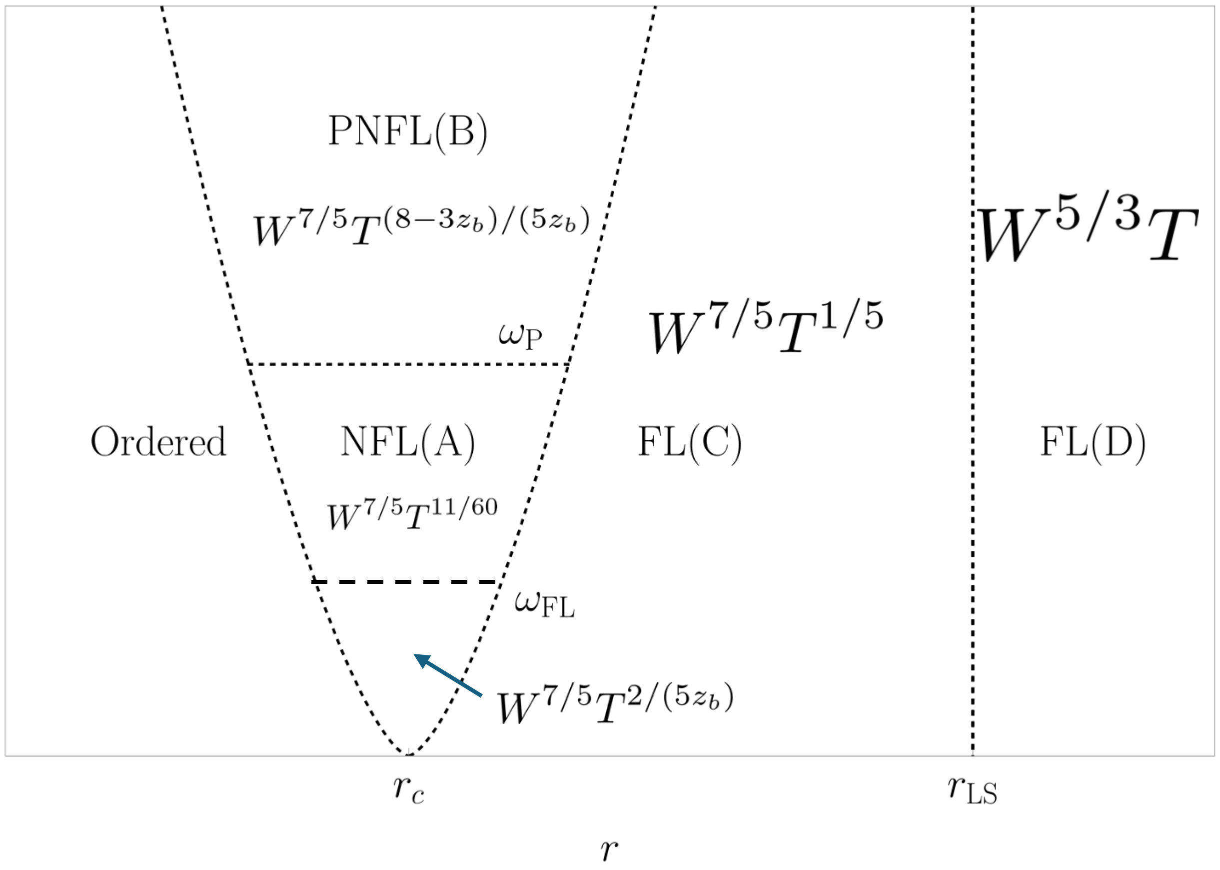


The different scaling exponents near the QCP can be experimental signature of a clean NFL

NFL Hydrodynamics

G(T,W) in tomographic regime

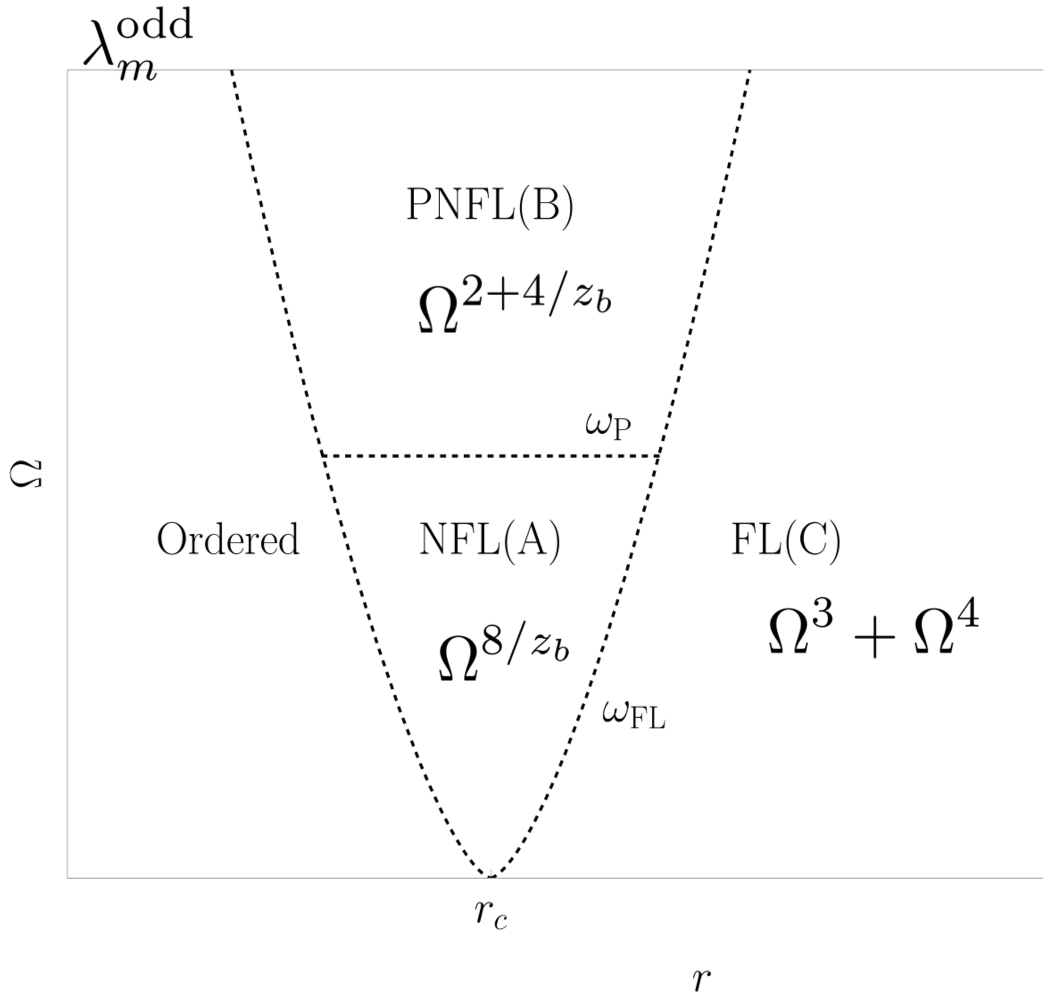
$l_{ee} < W < l_{tomo}$



There are two regimes in the strongly coupled region, depending on the type of dominating fluctuation:
Top: thermal fluctuation
Bottom: quantum fluctuation

Instability of the Ising-Nematic QCP

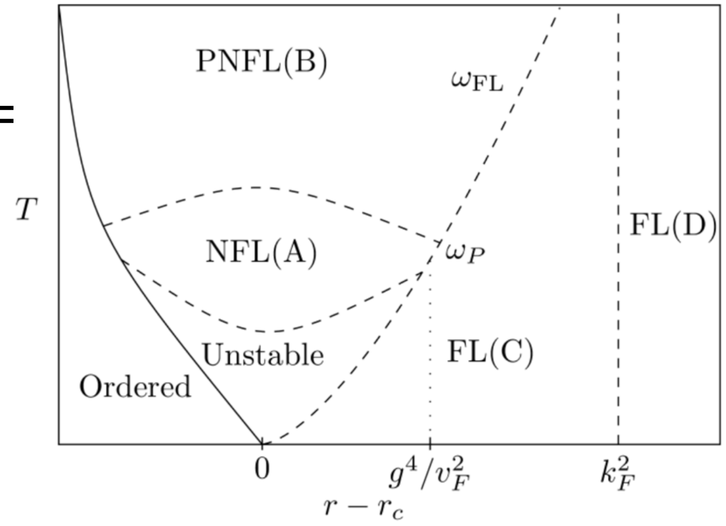
- Stability of the critical FS requires positive collision rates:
$$\Re\lambda_m(i\Omega \to \omega + i0) > 0$$
- All the even-m soft modes pass the stability test
- However, the odd-m modes of the NFL regime (A) is only stable if $\cos\frac{4\pi}{z_b} > 0$, i.e. $2 < z_b < 8/3$



Instability of the Ising-Nematic QCP

- For a translational-invariant, Q=0 , critical, convex FS, the NFL regime can be stable at T=0 only if $2 < z_b < 8/3$ (ignoring SC)
  - In particular, the $z_b = 3$ QCP from the is unstable at T=0
  - One possible numerical example is by Sam P. Ridgway and Chris A. Hooley [PRL 114, 226404 (2015)], who obtained $z_f = z_b /2 = 13/10$ from functional RG for a FM QCP
  - The NFL in Senthil's continuous Mott transition has $z_b = 2^+$, which falls into the stability bound
- At finite T, the thermal fluctuation can stabilize the theory

Proposed Phase diagram at z=3



Conclusions

- A clean metal (FL or NFL) can show anomalous transport behaviors due to hydrodynamics
- In the hydrodynamic regime, the interaction lubricates instead of impeding transport
  - The conductance $G(T, W)$ shows anomalous scaling
  - An experiment signature of clean NFL
- The NFL of the Ising-Nematic QCP is only stable at $T = 0$ when $2 < z_b < \frac{8}{3}$, or when $z_b \geq 3$ and at finite T